Notes – Determining if Lines are Parallel, Intersecting, or Coincident given points

I. **We can also determine if 2 lines are parallel, intersecting, or ____________ given 2 ____________ pairs for each line, or sometimes 1 ____________ pair and one ____________.

1) Does the line that passes through (0, 3) and (2, 7) intersect the line that passes through (5, 1) and (10, 2)?

| Steps: |
| 1) Find and compare the ________ of each line. |
| 2) If ________ are different, the lines are automatically ____________ lines. |
| 3) If ________ are the same, the lines could either be ____________ or ____________. |
| 4) If so, we’ll have to find the ________ by using the ________ formula and compare the two equations to determine if they’re parallel or coinciding. |

II. Are the following lines intersecting, parallel, or coinciding.

2) A line that passes through points (6, 9) and (0, -3) and a line that passes through points (4, 9) and (-2, -3).

3) A line that passes through (4, -1) and (-8, 6) and a line that passes through (12, -7) and (8, -4).

4) A line that has a slope of 3 and passes through (1, -1) and the line that passes through (1, 5) and (0, 2).
Coached Example

Does the line that passes through \((0, 2)\) and \((5, 5)\) intersect the line that passes through \((-10, -4)\) and \((-5, -1)\)? If not, are the two lines parallel or coincident?

Find the slope of the line that passes through \((0, 2)\) and \((5, 5)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{5} - 0 = \frac{3}{5}
\]

Find the slope of the line that passes through \((-10, -4)\) and \((-5, -1)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-4)}{-5 - (-10)} = \frac{3}{5}
\]

Are the slopes the same or different?
The slopes are \(\frac{3}{5}\). So, the lines are not intersecting lines.

To decide if the lines are parallel or coincident, compare their \(y\)-intercepts.

You know the first line passes through \((0, 2)\). That is its \(y\)-intercept.

Use the point-slope form to find the \(y\)-intercept of the other line.

The slope, \(m\), is \(\frac{3}{5}\). Let \((x_1, y_1) = (-5, -1)\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-1) = \left(\frac{3}{5}\right)(x - (-5))
\]

\[
y + 1 = \frac{3}{5}x + \_
\]

\[
y + 1 - 1 = \_
\]

\[
y = \_
\]

The equation above is in slope-intercept form.

Since \(b = \_
\)

the \(y\)-intercept of that line is \((0, \_
\)

Is that different or the same as the \(y\)-intercept of the first line?

The lines have the same \(\_
\) and \(\_
\), so they \(\_
\) intersect.

They are \(\_
\) lines.
I. **We can also determine if lines are parallel, intersecting, or coincident given points**.

1) Does the line that passes through \((0, 3)\) and \((2, 7)\) intersect the line that passes through \((5, 1)\) and \((10, 2)\)?

II. **Are the following lines intersecting, parallel, or coinciding?**

1) A line that passes through \((-2, 9)\) and \((0, -3)\) and a line that passes through \((4, 9)\) and \((2, 3)\).

2) A line that passes through \((6, 9)\) and \((0, -3)\) and a line that passes through \((4, 9)\) and \((2, 3)\).

3) A line that passes through \((4, 1)\) and \((8, 6)\) and a line that passes through \((4, 1)\) and \((8, 6)\).

4) A line that has a slope of \(3\) and passes through \((1, -1)\) and the line that passes through \((3, 1)\) and \((0, 2)\).

**Notes** - Determining if lines are Parallel, Intersecting, or Coincident given points.

**Steps:**

1) Find and compare the slope of each line.
2) If slopes are different, the lines are intersecting.
3) If slopes are the same, the lines could be either parallel or coinciding.
4) If so, we'll have to find the y-intercept by using the point-slope formula and compare the two equations to determine if they're parallel or coinciding.

Example:

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
\text{Since slopes are different, the lines are intersecting.}
\]

\[
y = mx + b
\]

\[
\text{Since slopes are different, the lines are parallel.}
\]

\[
\text{Since slopes are the same, the lines are coinciding.}
\]

\[
y = \frac{3}{4}x - \frac{9}{4}
\]

\[
y = \frac{3}{4}x - \frac{9}{4}
\]

\[
y = \frac{3}{4}x - \frac{9}{4}
\]
Coached Example

Does the line that passes through (0, 2) and (5, 5) intersect the line that passes through (−10, −4) and (−5, −1)? If not, are the two lines parallel or coincident?

Find the slope of the line that passes through (0, 2) and (5, 5).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{5 - 0} = \frac{3}{5} \]

Find the slope of the line that passes through (−10, −4) and (−5, −1).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-4)}{-5 - (-10)} = \frac{3}{5} \]

Are the slopes the same or different?

The slopes are \underline{same}. So, the lines are not intersecting lines.

To decide if the lines are parallel or \underline{coincident}, compare their \underline{y-intercepts}.

You know the first line passes through (0, 2). That is its \underline{y-intercept}. \[ b = 2 \]

Use the point-slope form to find the \underline{y-intercept} of the other line. (−10, −4) (−5, −1)

The slope, \( m \), is \( \frac{3}{5} \). Let \((x_1, y_1) = (−5, −1)\).

\[ y - y_1 = m(x - x_1) \]

\[ y - (-1) = \left(\frac{3}{5}\right)(x - (-5)) \]

\[ y + 1 = \frac{3}{5}x + \frac{15}{5} \]

\[ y + 1 - 1 = \frac{3}{5}x + \frac{3}{5} \]

Subtract 1 from both sides.

The equation above is in \underline{slope-intercept} form.

Since \( b = \frac{3}{5} \), the \underline{y-intercept} of that line is (0, \( \frac{3}{5} \)).

Is that different or the same as the \underline{y-intercept} of the first line? \underline{same}

The lines have the same \underline{slope} and \underline{y-intercept}, so they do not \underline{intersect}.

They are \underline{coincident} \underline{lines}.